

FUZZY REASONING INFERENCE ASSOCIATED WITH GENETIC ALGORITHM APPLIED TO A TWO-WHEELED SELF-BALANCING ROBOT OF LEGO MINDSTORMS NXT

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ABSTRACT

In this paper, we focus on a fuzzy knowledge-based reasoning control with genetic algorithm (GA) for a LEGO two-wheeled self-balancing robot. To achieve the control objectives with the satisfied states' responses, a GA is involved to search feasible solution sets of the state feedback gain. Based on the dynamic model provided by LEGO Mindstorms NXT, performance indices of systems' states can be characterized as rise time (RS), overshoot (OS), and integral of all error (IAE). Thus, a GA with a fitness-function embracing the three performance indices is thus proposed for searching a set of well response parameters. Furthermore, we employ fuzzy inference reasoning to blend multiple states' conditions. For demonstration, the proposed approach was compared with some existing works. By the simulation results, the proposed controller implemented by the fuzzy reasoning inference with GA reveals the well performances.

Keyword: Genetic Algorithm (GA), Fuzzy Inference, Two-Wheeled Robot, Performance Index.

1. INTRODUCTION

In recent years, the control of Two Wheeled Self-Balancing Robot (2WSBR) has been researched by many researchers (Salerno & Angels, 2007; Huang, Wang, & Chiu, 2011; Grasser, D'Arrigo, Colombi, & Rufer, 2002; Chen, Liu, Chen, & Lin, 2011; Xu, Guo, & Lee, 2013; Kanada, Watanabe, & Chen, 2011). Most research works on theoretical analysis and simulation results, still, some research works on implementation. Two wheeled mobile robot (2WMR) or Two wheeled inverted pendulum (2WIP) are common implementation of 2WSBR, such as JOE (Grasser et al., 2002), Quasimoro (Salerno & Angeles, 2007), and Segway (Segway, 2004). Another common implementation is using LEGO Mindstorms NXT kits (Chen et al., 2011; Kanada et al., 2011). Unlike the implementation of 2WMR and 2WIP, the model design and the application of LEGO is much easier and more diverse, for example, there were many research from education to control theory (Gawthrop & McGookin, 2004; Linda & Manic, 2011; Kim & Jeon, 2009; Kim, 2011). The controller design is important because the system is unstable without external force.

In plenty of stabilize algorithms (Ibanez, Frias, & Castanon, 2008; Pathak, Franch, & Agrawal, 2005), Lyapunov, active/passive, and state feedback control, etc., are based on the system model. In state feedback control, the feedback gain usually adjusted by linear quadratic regulator (LQR) with trial and error (Salerno & Angeles 2007; Yamamoto, 2008). Moreover, the external disturbance may be concerned because those stabilize algorithms proved based on the accurate mathematical model. Therefore, some researchers works on the controller which can provide additional information to the system (Huang et al., 2011; Chen et al., 2011; Xu et al., 2013; Kanada et al., 2011). To solve the problem of uncertainty, we adopted fuzzy reasoning which is widely used in many fields (Huang et al., 2011; J Chen et al., 2011; Xu et al., 2013). The knowledge-based design can make controller model free, even more, this kind of design can offer robustness in non-linear system (Xu et al., 2013). In general, fuzzy reasoning can be easily combined with a lot of controller due to its flexibility. Genetic Algorithm (GA), is a global search algorithm for finding well performance feedback gain in this work. This theory is inspired by natural evolution



Fig. 1. The structure of the 2WSBR

which has selection, crossover, and mutation as algorithm (Juang, Chang, & Huang, 2008). In this paper, for the performance consideration of balancing 2WSBR, we proposed a controller design strategy regarding the fuzzy reasoning inference with GA. According to different body slopping angles, fuzzy reasoning system will involve different feedback gain to the formulated dynamic model which is modified from the previous work (Yamamoto, 2008). Then, GA is applied for finding the better feedback gain of state feedback control. A presented fitness function can be formed by rise time (RS), overshoot (OS), and integral of all error (IAE) in the step response of the 2WSBR. The structure of this paper is as follows: The system description is presented in Section 2. In Section 3, the controller design is analyzed. Section 4, the simulation and implementation result will be mentioned. The conclusion is in the Section 5.

2. SYSTEM DESCRIBE

Fig. 1 shows the structure and system model of the proposed 2WSBR which is built by LEGO Mindstorms NXT kits. Proposed 2WSBR using LEGOs' DC motor which has wheel angle and wheel speed sensors. The gyro sensor of HiTechnic built for LEGO is used for measuring current body pitch angle. We change the data receiving from sensor into body pitch angle φ , body pitch angular velocity $\dot{\varphi}$, wheel angle θ , and wheel angular velocity $\dot{\theta}$ for system output. The coordinate system and parameters for 2WSBR shows in Fig. 2 and Tab. 1.

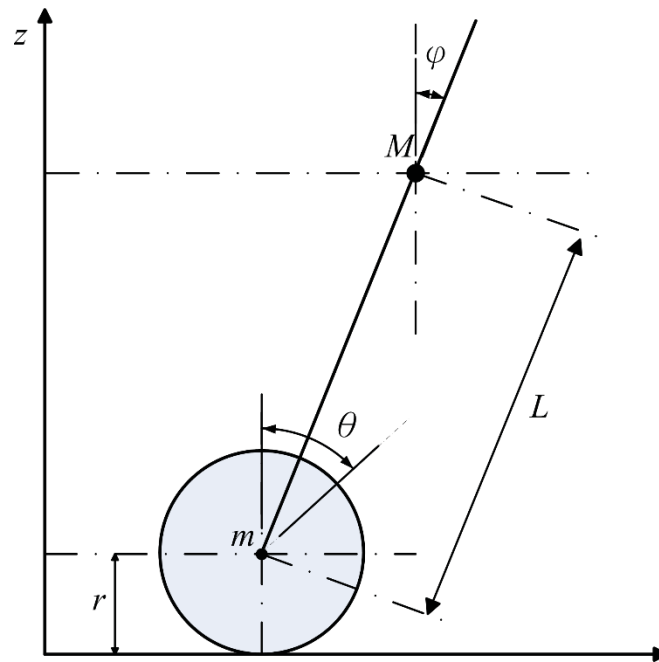


Fig. 2. The coordinate of the 2WSBR

2.1 System at balancing point

The derivation of the state equation of proposed 2WSBR is based on (Yamamoto, 2008). Following the Lagrange equations, we can have (1) and (2):

$$[(2m + M)r^2 + 2J_w + 2n^2J_d]\ddot{\theta} + (MLr \cos \varphi - 2n^2J_d)\ddot{\varphi} = F_\theta \quad (1)$$

$$(MLr \cos \varphi - 2n^2J_d)\ddot{\theta} + (ML^2 + J_b + 2n^2J_d)\ddot{\varphi} - MgL \sin \varphi = F_\varphi \quad (2)$$

Then, the generalized force can be expressed as (3) and (4):

$$F_\theta = \alpha(v_l + v_r) - 2(\beta + f_w)\dot{\theta} + 2\beta\dot{\varphi} \quad (3)$$

$$F_\varphi = -\alpha(v_l + v_r) + 2\beta\dot{\theta} - 2\beta\dot{\varphi} \quad (4)$$

Tab. 1. The parameters of 2WSBR

The mass of wheel	m	0.0168 (kg)
The mass of body	M	0.553 (kg)
The radius of wheel	r	0.056 (m)
The depth of body	D	0.094 (m)
The width of body	W	0.185 (m)
The distance between center of mass and wheel axle	L	0.082 (m)
The inertia moment of wheel	J_w	2.6×10^{-5} (kgm ²)
The inertia moment of body pitch	J_b	1.2×10^{-3} (kgm ²)
The inertia moment of DC motor	J_d	1×10^{-5} (kgm ²)
The resistance of DC motor	R_d	6.69 (Ω)
The back EMF constant of DC motor	K_e	0.468 (V sec/rad)
The torque constant of DC motor	K_t	0.317 (Nm/A)
Gravity	g	9.81 (m/sec ²)
Gear ratio	n	1
The friction coefficient between body and DC motor	f_d	0.0022
The friction coefficient between wheel and the ground	f_w	0

According to eq. (1) - (4), let the body pitch angle $\varphi \rightarrow 0$ ($\sin \varphi \rightarrow \varphi$ and $\cos \varphi \rightarrow 1$) and neglect the second order terms. Next, we let input $[v_l \ v_r]^T$ seems as synchronize, so we got u shows in (5):

$$\begin{bmatrix} e_{11} & e_{12} \\ e_{21} & e_{22} \end{bmatrix} \begin{bmatrix} \ddot{\theta} \\ \ddot{\varphi} \end{bmatrix} + \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix} \begin{bmatrix} \dot{\theta} \\ \dot{\varphi} \end{bmatrix} + \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} \theta \\ \varphi \end{bmatrix} = \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} u \quad (5)$$

where, $E = \begin{bmatrix} e_{11} & e_{12} \\ e_{21} & e_{22} \end{bmatrix}$, $F = \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix}$, $G = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix}$, $H = \begin{bmatrix} h_1 \\ h_2 \end{bmatrix}$,

$$e_{11} = (2m + M)r^2 + 2J_w + 2n^2J_d,$$

$$e_{12} = e_{21} = MLr - 2n^2J_d, e_{22} = ML^2 + J_b + 2n^2J_d$$

$$f_{11} = 2(\beta + f_w), f_{12} = f_{21} = -2\beta, f_{22} = 2\beta, g_{11} = g_{12} = g_{21} = 0, g_{22} = -MgL,$$

$$h_1 = \alpha, h_2 = -\alpha, \alpha = \frac{nK_t}{R_d}, \beta = f_d + \frac{nK_eK_t}{R_d}$$

Let $[\theta \ \varphi \ \dot{\theta} \ \dot{\varphi}]^T$ to be \mathbf{x} . The state equation (6) shows as following:

$$\dot{\mathbf{x}} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \mathbf{x} + \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} u, \mathbf{y} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} \\ c_{21} & c_{22} & c_{23} & c_{24} \\ c_{31} & c_{32} & c_{33} & c_{34} \\ c_{41} & c_{42} & c_{43} & c_{44} \end{bmatrix} \mathbf{x} + \begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \end{bmatrix} u \quad (6)$$

$$\text{where, } A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix}, B = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix},$$

$$C = \begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} \\ c_{21} & c_{22} & c_{23} & c_{24} \\ c_{31} & c_{32} & c_{33} & c_{34} \\ c_{41} & c_{42} & c_{43} & c_{44} \end{bmatrix}, D = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \end{bmatrix},$$

$$\det(E) = e_{11}e_{22} - e_{12}e_{21},$$

$$a_{32} = \frac{g_{22}e_{12}}{\det(E)}, a_{42} = -\frac{g_{22}e_{11}}{\det(E)}, a_{33} = -\frac{f_{11}e_{22}}{\det(E)} + \frac{f_{12}e_{12}}{\det(E)}, a_{43} = \frac{f_{11}e_{12}}{\det(E)} + \frac{f_{22}e_{11}}{\det(E)},$$

$$a_{34} = \frac{f_{22}e_{22}}{\det(E)} + \frac{f_{22}e_{12}}{\det(E)}, a_{44} = \frac{f_{12}e_{11}}{\det(E)} + \frac{f_{12}e_{12}}{\det(E)},$$

$$a_{11} = a_{21} = a_{31} = a_{41} = 0, a_{12} = a_{14} = a_{22} = a_{23} = 0,$$

$$b_3 = \frac{h_1e_{22}}{\det(E)} + \frac{h_1e_{12}}{\det(E)}, b_4 = \frac{h_2e_{11}}{\det(E)} + \frac{h_2e_{12}}{\det(E)}, b_{11} = b_{12} = b_{21} = b_{22} = 0,$$

$$C = \text{Identity Matrix}, D = \text{Zero Matrix}$$

Therefore, the state function of system at balancing point can be expressed by (7):

$$\dot{\mathbf{x}} = A\mathbf{x} + Bu, \quad \mathbf{y} = C\mathbf{x} + Du \quad (7)$$

2.2 System at different point

In proposed system, we analyzed the different body pitch angle of 2WSBR. First, the body pitch angle has been divided to 0, ± 5 , ± 10 , ± 15 , ± 20 , ± 25 , and ± 30 degrees, where $\varphi = \frac{i\pi}{36}$, $i = -6, -5, \dots, 6$. Second, the new state equation will be derived with those divided point. Third, calculate the feedback gain for each point of states.

Due to the different points, we have to adjust the body pitch angle of original state equation which is seems as stable state. We adjust φ to be different angle shows in

Tab. 2:

Tab. 2. Body pitch angle

φ (deg.)	0	± 5	± 10	± 15	± 20	± 25	± 30
$\sin \varphi$	φ	0.9989φ	0.9948φ	0.9885φ	0.9797φ	0.9778φ	0.9549φ
$\cos \varphi$	1	0.9962	0.9848	0.9659	0.9397	0.9063	0.866

Next, we applied those parameters into (1) and (2) so that we can get state equations for each point, for example, after applied the parameters of degree 5 of body pitch angle, e_{12} and e_{21} become $0.9962MLr - 2n^2J_d$, g_{22} become $-0.9989MgL$. Following state equations adjust eq. (7) shows the body pitch angle for 0, ± 5 , ± 10 , ± 15 , ± 20 , ± 25 , and ± 30 degrees:

$$\dot{x} = A_i x + B_i u \quad (8)$$

where $i = 1, 2, \dots, 13$. A_1, B_1 stands for the state equation of 30 degrees. A_2, B_2 stands for the state equation of 25 degrees. A_3, B_3 stands for the state equation of 20 degrees. A_4, B_4 stands for the state equation of 15 degrees. A_5, B_5 stands for the state equation of 10 degrees. A_6, B_6 stands for the state equation of 5 degrees. A_7, B_7 stands for the state equation of 0 degrees. A_8, B_8 stands for the state equation of -5 degrees. A_9, B_9 stands for the state equation of -10 degrees. A_{10}, B_{10} stands for the state equation of -15 degrees. A_{11}, B_{11} stands for the state equation of -20 degrees. A_{12}, B_{12} stands for the state equation of -25 degrees. A_{13}, B_{13} stands for the state equation of -30 degrees.

2.3 Fuzzy applied to the system

We applied the fuzzy reasoning into the system which shows in (8). This kind of fuzzy reasoning needs to know body pitch angle φ , thus, we can have rules of the model:

$$R^i: \text{IF } x_2 \text{ is } M^i \text{ THEN } \dot{x} = A_i x + B_i u \quad (9)$$

where $i = 1, 2, \dots, 13$, M^i is the fuzzy labels, u is the volt of system as control input which is going to define in the next section.

By using fuzzy labels M^i , negative big big (NBB), negative big small (NBS), negative medium big (NMB), negative medium small (NMS), negative small big (NSB), negative small small (NSS), zero (ZO), positive small small (PSS), positive small big (PSB), positive medium small (PMS), positive medium big (PMB), positive big small (PBS), and positive big big (PBB), the inputs are associated with these

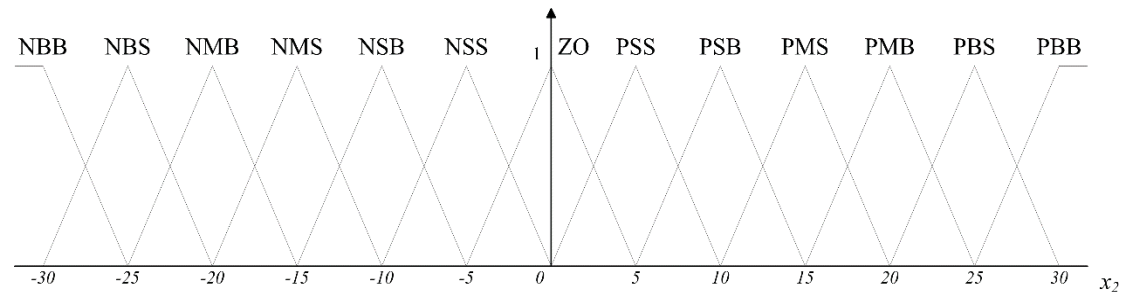


Fig. 3. The membership function for fuzzy reasoning

respectively. The triangular membership function shows in Fig. 3, Therefore, the output of the fuzzy reasoning of system is as following weighted form:

$$\dot{\mathbf{x}} = \frac{\sum_{i=1}^{13} M^i(x_2)(A_i \mathbf{x} + B_i u)}{\sum_{i=1}^{13} M^i(x_2)} \quad (10)$$

Due to the symmetry of membership function $\sum_{i=1}^{13} M^i(x_2)$ equals to 1, so we have:

$$\dot{\mathbf{x}} = \sum_{i=1}^{13} M^i(x_2)(A_i \mathbf{x} + B_i u) \quad (11)$$

In next section, the proposed system (11) which applied fuzzy reasoning will be used as the model for control method.

3. CONTROLLER DESIGN

In this section, we are going to describe the proposed control method. First, the state feedback control is introduced. Next, using fuzzy reasoning to handle the error of the state feedback. Then, the feedback gain which is tuned by genetic algorithm (GA) will be applied to the system.

3.1 State feedback control

The state equation mentioned in (7) and (8) can be tuned by adjusting the feedback gain $\mathbf{k} \in R^{i \times 4}$ of the state feedback control.

$$u = -kx \quad (12)$$

Therefore, we can replace (12) into (13):

$$\begin{aligned} \dot{x} &= (A - Bk)x \\ y &= (C - Dk)x \end{aligned} \quad (13)$$

The system can be stabilized by adjusting feedback gain k due to the eigenvalues of the $A - Bk$ (Yamamoto, 2008). Therefore, the proposed fuzzy reasoning associate with GA will be applied to 2WSBR in following sections.

3.2 Fuzzy reasoning

The objective of the proposed 2WSBR is to ensure the stability of the system. Based on (9)-(13), we applied the T-S-type fuzzy reasoning to proposed 2WSBR. With state feedback, the IF-THEN rules can be expressed as following form:

$$R^i: \text{IF } x_2 \text{ is } M^i \text{ THEN } u = -k_i x \quad (14)$$

where $i = 1, 2, \dots, 13$, M^i is the fuzzy labels, k_i is the i th feedback gain. The final out of fuzzy reasoning is expressed as

$$u_{Fuzzy} = - \frac{\sum_{i=1}^{13} (M^i(x_2)(k_i x))}{\sum_{i=1}^{13} M^i(x_2)} = - \sum_{i=1}^{13} (M^i(x_2)(k_i x)) \quad (15)$$

Tab. 3 shows the 13 rules of the fuzzy reasoning. The feedback gain k_i will be applied due to the different error of body pitch angle so that the value of k_i has symmetric look in the rule table.

According to (11) and (15), the fuzzy reasoning system model can be combined by replacing the u in (11). Next, the adjustment of feedback gain k will be proposed.

Tab. 3. Fuzzy rule table

Rule	$x_2(\varphi)$	$u(-k_i x)$
1	PBB	$-k_1 x$
2	PBS	$-k_2 x$
3	PMB	$-k_3 x$
4	PMS	$-k_4 x$
5	PSB	$-k_5 x$
6	PSS	$-k_6 x$
7	ZO	$-k_7 x$
8	NSS	$-k_8 x = -k_6 x$
9	NSB	$-k_9 x = -k_5 x$
10	NMS	$-k_{10} x = -k_4 x$
11	NMB	$-k_{11} x = -k_3 x$
12	NBS	$-k_{12} x = -k_2 x$
13	NBB	$-k_{13} x = -k_1 x$

3.3 Choosing feedback gain by genetic algorithm

Genetic algorithm (GA) is a global search algorithm which mimics the natural evolution. This paper works on finding a better feedback gain using GA. The

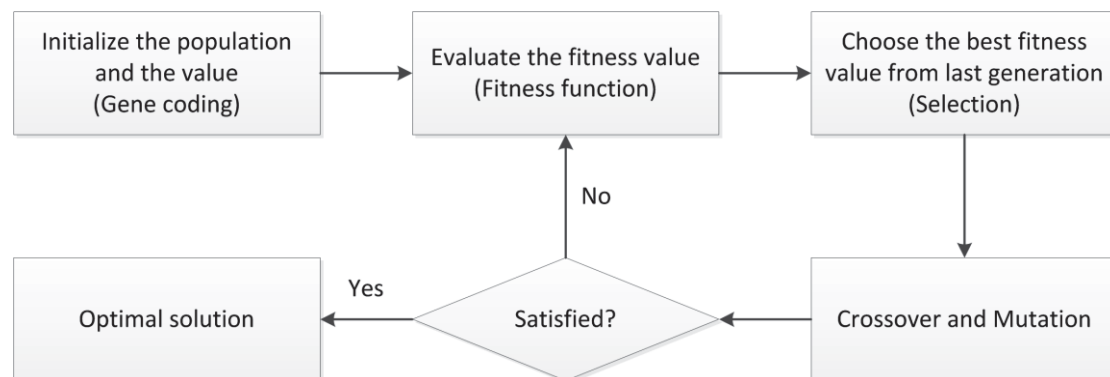


Fig. 4. The flow chart of GA

performance indices of the step response of the 2WSBR can be defined with rise time (RS), overshoot (OS), and integral of all error (IAE). The fitness function of GA is defined with those three parameters.

$$f_{sys} = fit(sys) = e^{-w_1 \cdot sys(RS)} \times e^{-w_2 \cdot sys(OS)} \times e^{-w_3 \cdot sys(IAE)} \quad (15)$$

where $w_1 \cdot w_2 \cdot w_3$ stands for the weight of RS, OS, and IAE, $sys(RS) \cdot sys(OS) \cdot sys(IAE)$ stands for the value of RS, OS, and IAE of the system. As mention before, we can define the minimum and maximum value of feedback gain shows in Tab. 4 by the eigenvalue of system matrix $A - Bk$. The feedback gain is defined as $k = [k_1 \ k_2 \ k_3 \ k_4]$.

Tab. 4. The minimum and maximum value of feedback gain

	k_1	k_2	k_3	k_4
Minimum	-4	-60	-3	-5
Maximum	-0.5	-10	-0.5	-1

4. SIMULATION AND IMPLEMENTATION RESULT

In this section, we have simulation and implementation for the proposed method: fuzzy knowledge-based reasoning control with GA. First, the existing works with fixed feedback gain will be applied to the different dynamic model. Second, the different feedback gain which is adjusted by GA will be compared with the feedback gain provided by existing works. Third, the performance of simulation and implementation results with proposed fuzzy reasoning system.

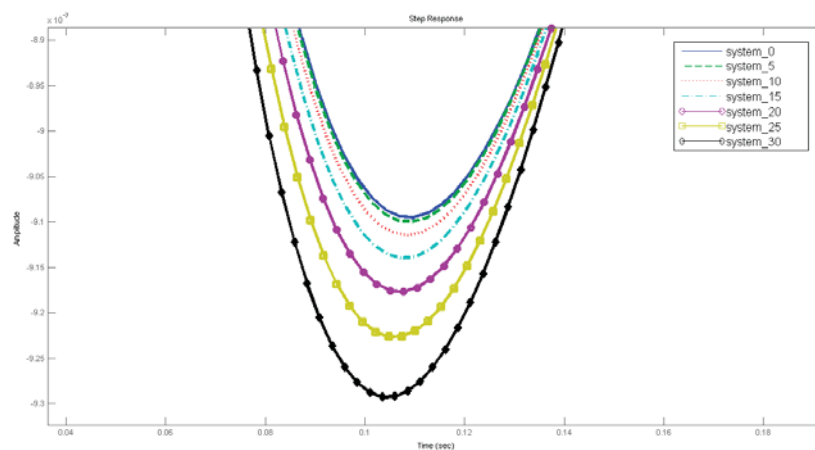


Fig. 5. The step response of fixed feedback gain

4.1 Fixed feedback gain

The overshoot (OS) of step response of fixed feedback gain adjusted by GA is shown in Fig 5. As mentioned before, the model of different body pitch angle has been derived. When we applied the fixed feedback in to the models, the OS is getting bigger. The OS value of each model of 0, 5, 10, 15, 20, 25, and 30 degrees are 9.0932×10^{-3} , 9.0979×10^{-3} , 9.1127×10^{-3} , 9.1402×10^{-3} , 9.1770×10^{-3} , 9.2270×10^{-3} ,

and 9.2932×10^{-3} respectively.

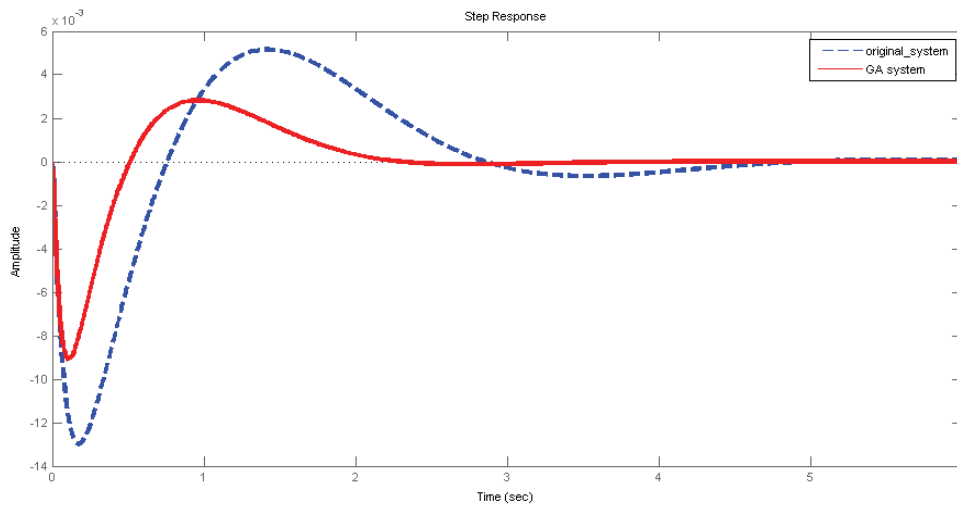


Fig. 6. The step response of fixed feedback gain

4.2 Feedback gain of each model

We applied the feedback gain which is adjusted by linear quadratic regulator (LQR) to the system at 0 degrees. Then the feedback gain adjusted by GA is compared with the feedback gain adjusted by LQR is shown in Fig 6. Fig 7 shows that the feedback gain of each model which is adjusted by GA. Now, we can see that the OS of the step response of each model is getting smaller. The OS value of each model of 0, 5, 10, 15, 20, 25, and 30 degrees are 9.0932×10^{-3} , 8.3337×10^{-3} , 8.0919×10^{-3} , 8.0579×10^{-3} , 8.0339×10^{-3} , 8.0712×10^{-3} , and 7.9741×10^{-3} respectively.

Tab. 5. The value of feedback gain

	k_1	k_2	k_3	k_4
System_0	-2.2342	-49.8668	-1.9154	-3.9916
System_5	-2.2670	-51.2496	-1.9972	-4.9512
System_10	-2.4115	-51.6881	-1.9850	-5.4137
System_15	-2.4324	-51.9500	-1.9896	-5.4492
System_20	-2.4442	-52.0906	-1.9885	-5.4952
System_25	-2.4872	-52.1179	-1.9986	-5.4629
System_30	-2.4999	-53.1457	-1.9996	-5.4942

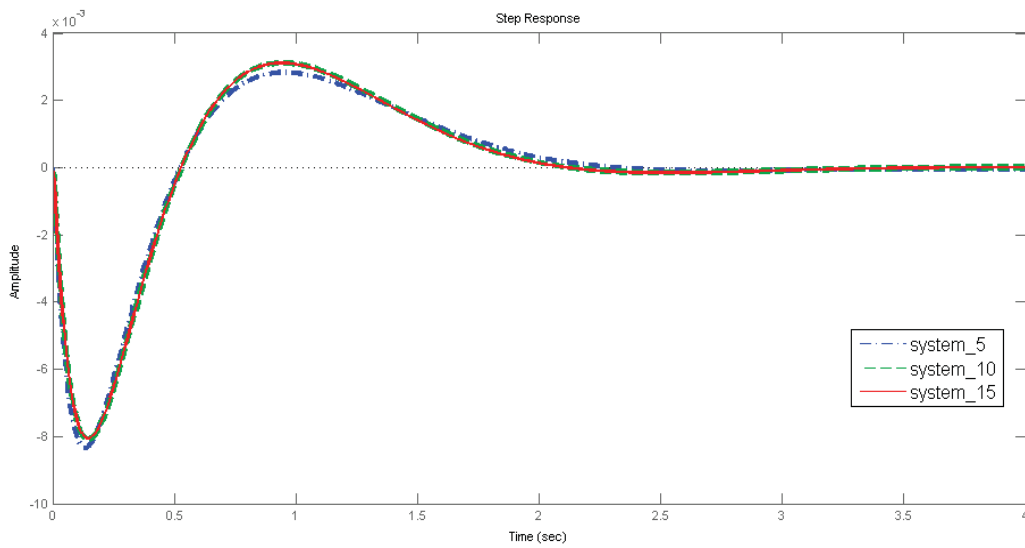


Fig. 7. (a). The step response of different feedback gain of model 5, 10, and 15.

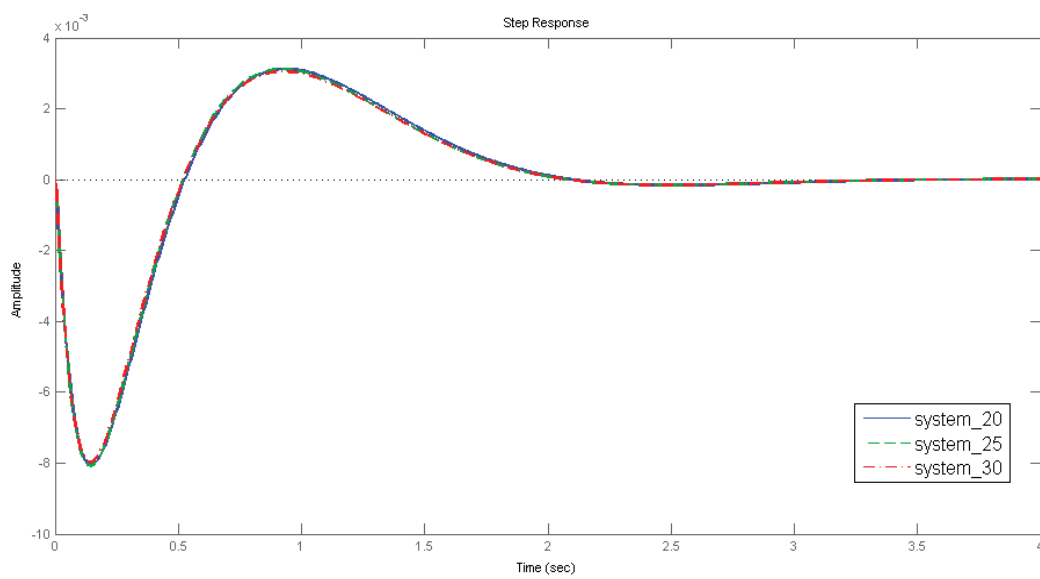


Fig. 7. (b). The step response of different feedback gain of model 20, 25, and 30.

4.3 Fuzzy reasoning applied to the model

With the feedback adjusted by GA which shows in Tab. 5, we applied the fuzzy reasoning by (11) and (15). Then, we assumed that the body pitch angle is at 7 degree in Fig 8. Fig 9 has a closer look at the point of OS.

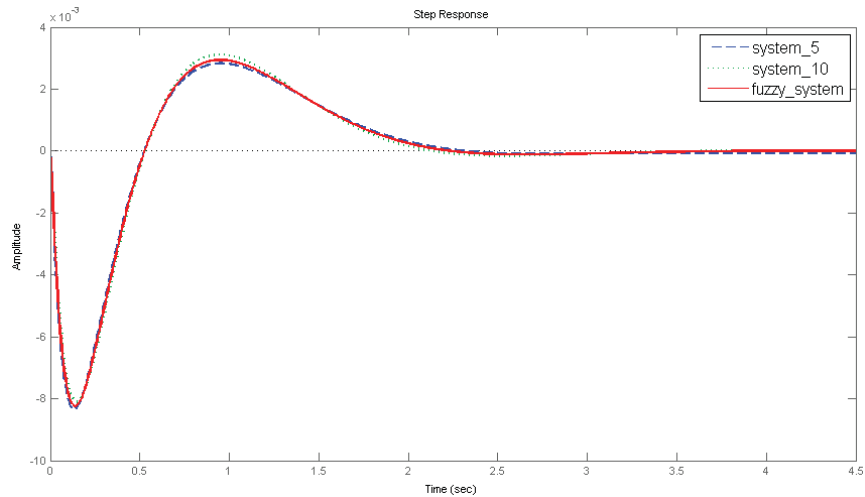


Fig. 8. The step response of fuzzy reasoning system at 7 degree

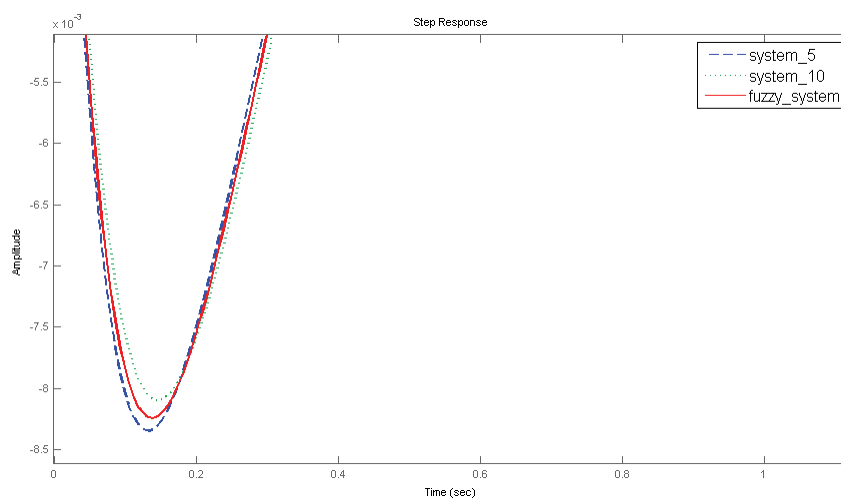


Fig. 9. The OS of step response of fuzzy reasoning system at 7 degree

With the proposed fuzzy reasoning control with GA for the 2WSBR, the performance of system with RS, OS, and IAE is better than exist work shows in Fig 6 – Fig 9. Even more, the proposed fuzzy reasoning makes the 2WSBR has a better performance.

5. CONCLUSION

In this work, the fuzzy knowledge-based reasoning control with GA for the 2WSBR of LEGO Mindstorms NXT has been proposed. To achieve the control objectives with the satisfied states' responses, a GA is involved to search feasible state feedback gains. Based on the dynamic model provided by LEGO Mindstorms NXT, we chose rise

time (RS), overshoot (OS), and integral of all error (IAE) of systems' step response as performance indices. Furthermore, with fuzzy reasoning, different feedback gain will be applied to the formulated dynamic model according to different body slopping angle.

With the simulation results, we can notice that the states' response of 2WSBR is not only has a better performance but also has a better response with different body slopping angle. The proposed fuzzy knowledge-based reasoning control with GA reveals the better performances.

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