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Systematically Description of Yrast Superdeformed Even-Even Hg Isotope Nuclei

Lung-Ming Chen*

ABSTRACT

It is shown that the Project Shell Model may present a systemic description of the yrast superdeformed bands in even-even Hg isotopes nuclei. The calculated γ-ray energies are compared with data for which spin is unambiguously assigned. Excellent agreement with available data for all isotopes is obtained. The calculated electromagnetic properties provide a microscopic understanding agree with those measured ones.

Key words: nuclear structure theory, extended shell model, projected shell model.

* Professor, Department of Computer Science and Information Engineering, Sue-Te University

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INTRODUCTION

Models, based on the Projected Shell Model (PSM) (Hara and Iwasaki, 1980, Hara and Sun, 1991) to describe the nuclear Superdeformed structure phenomena, has been applied extensively and make very successful in mass A~60, 130 and 190 regions (Sun and Guidry, 1995, Sun, Velazquez, Hirseh, and Sun, 1998, Sun, Zhang, and Guidry, 2002, Guo and Chen, 2003). The experimental data on superdeformed bands (SD) are given in the form of a series of γ-ray energies but only a few cases have the spins in an SD band been determined directly by experiment. ^{194}Hg is the one of only two SD nuclei in the mass A~190 region that the assigned spin of each γ-ray energy have been measured with high precision (Han and Wu, 1998). In addition, their transition quadrupole moments have also obtained from the lifetime measurements (Han and Wu, 1998). However, level spins in most of these neighboring nuclei were not determined due to the linking transitions between SD band and Normal-deformed band were not observed experimentally. Several models have been proposed for the spin assignments in these SD bands (Wu et al, 1992) but the model dependence of these theoretical spin determinations is not accurate enough to make a spin assignment within the estimate error . However, it is delighted that the spin determination studies for the SD states gave a bright light on the structure of these states (Hara and Sun, 1995). The PSM is a shell model truncated in a deformed (Nilsson-type) single particle basis, with pairing correlation incorporated into the basis by a BCS calculation for the Nilsson states. More precisely, the truncation is first implemented in the multi-quasiparticle basis with respect to the deformed BCS vacuum state; then the violation of rotational symmetry is removed by projection to form a shell model basis in the laboratory frame. Finally a shell model Hamiltonian is diagonalized in the projected space. In this paper, a systemic description of the SD bands in even-eve $^{188\sim196}Hg$ isotopes based on the PSM is presented.

MODEL

For the present study, we chosed the multi-quasiparticle (qp) states, $|\Phi_\kappa\rangle$, as

$$\{|0\rangle, \alpha^+_{n_i}\alpha^+_{n_j}|0\rangle, \alpha^+_{p_m}\alpha^+_{p_n}|0\rangle, \alpha^+_{n_i}\alpha^+_{n_j}\alpha^+_{p_m}\alpha^+_{p_n}|0\rangle\} \quad (1)$$

for double even nuclei. Where α^+ is the creation operator for a quasi-particle and the index n (p) denotes neutrons(protons). The many-body wave function is a superposition of projected (angular momentum) multi-quasiparticle states,

$$|\Psi^I_M\rangle = \sum_{\kappa K} f^I_{\kappa K} P^I_{MK} |\Phi_\kappa\rangle \quad (2)$$

where P^I_{MK} is the angular momentum projection operator and the coefficients $f^I_{\kappa K}$ are the variation parameters. We use the usual separable-force Hamiltonian (Hara and Sun 1995)

$$H = \hat{H}_0 - \tfrac{\chi}{2}\sum_\mu \hat{Q}^+_\mu \hat{Q}_\mu - G_M \hat{P}^+ \hat{P} - G_Q \sum_\mu \hat{P}^+_\mu \hat{P}_\mu \quad (3)$$

which has been used successfully to explain the system of the rotational spectra for a large number of nuclei. The first term is the spherical single particle Hamiltonian,

$$\hat{H}_0 = \sum_\alpha c^+_\alpha E_\alpha c_\alpha \quad (4)$$

where c^+_α, c_α is the single particle creation and annihilation operator respectively and E_α is the single particle energy

$$E_{\alpha} = \hbar\omega[N - 2\kappa\hat{l} \bullet \hat{s} - \kappa\mu(\hat{l}^2 - \langle \hat{l} \rangle^2)], \qquad (5)$$

where ω is the harmonic-oscillator parameter which incorporates the principle of volume conservation for nuclei deformed form spherical shapes. The s and l represents the intrinsic nucleon spin and orbital angular momentum in the stretched coordinate basis. For the Nilsson parameters κ and μ are taken from the N-dependent values in Bengtsson and Ragnarssio (1985), subject to modifications introduced by Zhang (1989). The value of deformation parameter ε_2 in the Nilsson model will affect the energy gap and the selection of quasi-particle basis. Normally the values of ε_2 were set according to the experimental observation if available or the same as the nearest isotope otherwise. Since the deformation parameters were well-studied quantity for these nuclei with the range from 0.43 to 0.48 accordingly (Han and Wu, 1998). Adjusting these parameters slightly was not affect the final results significantly in our calculation. Thus, instead of deriving it from self-consistent mean-field calculations, we consider it to be known and use a value of 0.455 for all nuclei. The remaining terms in Eq. (3) are residual quadrupole-quadrupole, monopole pairing, and quadrupole pairing interaction, respectively. The strength of the quadrupole-quadrupole force was fixed to give the measured deformation of SD band. The strength of the monopole pairing interaction is critical for a quantitative discussion of the moment of inertia. Usually the incorporation of high-N shells in the valence space is important for the SD moment of inertia, thus the single-particle configurations are consisted of three major shells each for neutrons (N=5, 6 and 7) and protons (N=4, 5 and 6) in our calculation. The total dimension of qp basis in our case is 50. The pairing strength is a function of the size of single-particle space and decrease roughly as the inverse square root of the participating levels in the BCS correlation . The prescription introduced in Sun and Guidry (1995) of,

$$G_M^n = (20.0 - 14.4\frac{N-Z}{A})A^{-1} \qquad (6)$$

and

$$G_M^p = 19.6 A^{-1} \qquad (7)$$

were used in our calculation, where n(p) is denoted for neutrons (protons). The strength of the quadrupole pairing force G_Q in Eq.(3) is assumed to be proportional to the monopole pairing G_M and will affect the spin value of the band crossing. One may carefully adjust the ratio of G_Q / G_M during the calculation to get the best representation of experimental observation. In our case, the ratio is set to 0.3 after comparing with the well measured ^{194}Hg and is used for all nuclei in this calculation. After diagonalizing the Hamiltonian in the quasi-particle basis, the lowest energy for each spin are used to compare with the experimental SD yrast energy. The resulting wave functions are usually used to compute the transition quadrupole moment or the gyromagnetic factor if necessary. Since the wave functions are eigenfunctions of angular momentum, we can calculate unambiguously the transition quadrupole moment Q_t as function of angular momentum. Usually, in the case of rigid rotor, the Q_t is constant as function of spin. When the band crossing occurs, the Q_t values are changed due to the small overlapping of the wave functions and bring about the shape changed in nuclei (Guo and Chen, 2003). In this calculation, we have used the relation between Q_t(I) and B(E2) transition probability through

$$Q_t(I) = \left(\frac{16\pi}{5} \frac{B(E2, I \rightarrow I-2)}{\langle IK, 20 | I-2K \rangle}\right)^{1/2} \qquad (8)$$

Where we use the same effective charges of 1.5e for protons and 0.5e for neutrons as in the previous PSM calculations(Guo and Chen, 2003, Sun, Zhang, and Guidry 1997).

RESULTS

The calculations were started with the well measured nucleus, ^{194}Hg, for a trial and then followed by the sequences of $^{188\sim196}Hg$ isotopes without adjusting any parameter. The calculated $E_\gamma, J^{(1)}$ and $J^{(2)}$ are defined as

$$J^{(1)} = [(2I-1)/E_\gamma(I)](\hbar^2 MeV^{-1}),$$

and (9)

$$J^{(2)} = 4/[E_\gamma(I+2) - E_\gamma(I)](\hbar^2 MeV^{-1})$$

here the γ-ray energies are defined as $E_\gamma = E(I) - E(I-2)$ and with the unit in MeV. Our calculations are compared with the experimental data (Han and Wu, 1998) and described on figure. The agreement between our calculations and experimental data are pretty good. The moment of inertia, $J^{(2)}$ rises gradually with the angular momentum for all nuclei and with various increase rate from nucleus to nucleus. The different rising slope has been indicated in Guo, Chou, and Chen (2004) as arise from the variation with particle number of effective interaction strengths and band crossing for different nuclei is also reproduced well in our calcul$\hbar$ions. Especially the observed downturn behavior in $J^{(2)}$ for ^{194}Hg at higher spin regions, the whole $J^{(2)}$ pattern forms a smooth bump with maximum at I = 44η. At this spin the SD ground-state band crosses, which is originated from the ground-state band crossing as suggested at $\hbar$ Guo, Chou, and Chen (2004). Rather constant Q_t distributions for these nuclei were found in our calculations, except a smoothly decrease at higher spin. For ^{194}Hg we have a value 16.9 eb up to I = 30η that is comparable to the measured average Q_t of 17.2 ± 2.0 eb. Above this spin, the theoretical values are slowly quenched. This is associated with gradual alignment processes that lead to a small reduction of collectivity (Guo, Chou, and Chen, 2004).

CONCLUSION

We have systematically presented the calculations of even-even $^{188\sim196}Hg$ SD bands via using the projected shell models with the same footing. The gradual increase of moment of inertia for double even SD nuclei in the Hg isotope region is due to the smoothly quenching of pair correlations by the Coriolis anti-pairing effect and the gradual rotational alignment of high-j quasi-particles. Hg nuclei have 80 protons and its ground state lies just below the shell-closure, at higher spins the proton pair correlations have higher possibility to be break and the align process will decrease the single neutron blocking effect. This results the slowly increasing of the moment of inertia. The PSM give us a microscopic understanding about the Hg double even SD nuclei in general.

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汞同位素偶－偶原子核之基態超形變的系統化記述

陳龍銘*

摘　要

本文係以投射殼層模型對汞同位素偶－偶原子核之基態超形變的能帶作系統化的研究，計算求得的 γ 射線能量與同位素中已確定自旋之實驗值非常吻合，其他求得的電磁性質亦與實驗值十分吻合。

關鍵字：原子核結構理論、殼層模型的擴展、投射殼層模型

* 樹德科技大學資訊工程學系教授